INDIAN SCHOOL NIZWA - WORKSHEET

APPLIED MATHEMATICS

3a. DERIVATIVES AND ITS APPLICATIONS

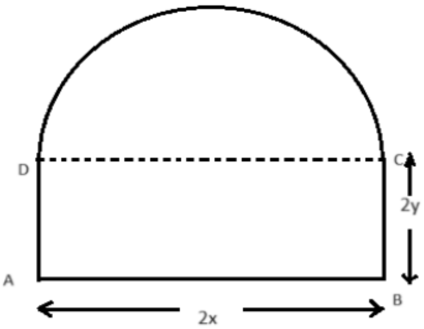
Name: ____________________ Date:_____________ Class: XII Sec: ___

1.	**1. If x – y = π, then $\frac{dy}{dx}$ =** **a) 0 b) 1 c) -1 d) 2**
2.	**If x = 4t and y = $\frac{4}{t}$, then $\frac{dy}{dx}$ =** **a. $\frac{1}{t^2}$ b. $\frac{-1}{t^2}$** **c. $\frac{2}{t^2}$ d. $\frac{-2}{t^2}$**
3.	**If e^y(x + 1) = 1, then which of the following is true:** **a. $\frac{d^2 y}{dx^2} = (\frac{dy}{dx})^2$ b. $\frac{d^2 y}{dx^2} = \frac{dy}{dx}$** **c. $(\frac{d^2 y}{dx^2})^2 = (\frac{dy}{dx})^2$ d. $\frac{d^2 y}{dx^2} = \frac{dy}{dx}$**
4.	**A toy manufacturing firm assesses its variable cost to be 'x' times the sum of 30 and 'x', where 'x' is the number of toys produced, also the cost incurred on storage is ₹ 1500. Then the marginal cost when 20 toys are produced is** **a. 60 b.70 c. 40 d. 15**
5.	**Find the intervals in which the functions f(x) = x^2 – 4x + 6 is strictly increasing** **(a) $(-\infty, 2) \cup (2, \infty)$ (b) $(2, \infty)$** **(c) $(-\infty, 2)$ (d)$(-\infty, 2] \cup [2, \infty)$**
6.	**The intervals in which $y = x^2e^{-x}$is increasing** **(a) $(-\infty, \infty)$ (b) (– 2, 0)** **(c) (2, ∞) (d) (0, 2)**

7.	**The curve $y = x^{\frac{1}{5}}$ has at (0, 0)** **(a)A vertical tangent (parallel to y – axis)** **(b) A horizontal tangent (parallel to x – axis)** **(C)An oblique tangent** **(d)No tangent**
8.	**At what point the slope of tangent to the curve $x^2 + y^2 – 2x – 3=0$ is zero ?** (a) **(3,0), (-1, 0)** **(b) (3, 0), (1, 2)** **(c) $(-1, 0), (1, 2)$** **(d) $(1, 2), (1, -2)$**
9.	**The cost function of a firm is $C = 3x^2 + 2x – 3$. The marginal cost , when x = 3 is:** **(a)10** **(b)25** **(c) 5** **(d) 20**
10.	**The function $f(x) = x^x$ has a stationary point at** **(a) x = e** **(b) $x = \frac{1}{e}$** **(c) x = 1** **(d) $x = \sqrt{e}$**
11.	It is given that at x = 1, the function $f(x) = x^3 – 12x^2 + kx + 7$ attains maximum value, then the value of 'k' (a) 10 (b) 12 (c) 21 (d) 13
12.	The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$ is (a) $(3)^{\frac{1}{3}}$ (b) $\frac{1}{2}$ (c) 1 (d) 0
13.	The normal to the curve $x^2 = 4y$ passing through (1, 2) is (a) x + y = 3 (b) x – y = 3 (c) x + y = 1 (d) x – y = 1
14.	Side of an equilateral triangle expands at a rate of 2cm/sec. the rate of increase of its area when each side is 10 cm is:

	(a) $10\sqrt{2}$ (b) $10\sqrt{3}$ (C)10 (d)5
15.	If y $=\frac{4}{x^2}+\sqrt{x}-\frac{1}{\sqrt{x}}$ then y'= ? 1) $\frac{8}{x^3}+\frac{2}{\sqrt{x}}+\frac{2}{x^{3/2}}$ 2) $-\frac{8}{x^3}+\frac{1}{2\sqrt{x}}+\frac{1}{2x^{3/2}}$ 3) $-\frac{8}{x^3}+2\sqrt{x}+2x^{3/2}$ 4) None of these
16.	**Assertion (A):** The maximum profit that a company makes if profit function is given by $P(x) = 41 + 24x - 8x^2$; where 'x' is the number of units and P is the profit is 59 **Reason (R)** : The profit is maximum at x = a if $P'(a) = 0$ *and* $P''(a) > 0$
17.	**Assertion (A):** If $y = xe^x$ *then* $\frac{dy}{dx} = xe^x + e^x$ **Reason(R):** $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
18.	**Assertion(A):** The price per unit of a commodity produced by a company is given by p=30-2x and 'x' is the quantity demanded. The marginal revenue when 5 commodities are in demand (or produced) is 10 **Reason(R):** Marginal revenue when x= 5 is $\frac{d\ (30x-2x^2)}{dx}$ at x = 5 is 10
19.	**Assertion (A) :** $f(x) = logx$ is defined for all $x\in(0, \infty)$. **Reason (R) :** If $f'(x) > 0$, then f(x) is strictly increasing function
20.	**Assertion (A)** : For the curve $x^3 + y^3 = 6xy$, the slope of the tangent at (3, 3) is 2. **Reason (R)** :The$\left(\frac{dy}{dx}\right)_{at\ (x_1,y_1)}$ gives slope of tangent of y = f(x) at (x_1, y_1).
21.	**Assertion (A)**: Equation of tangent at the point (2, 3) on the curve $y^2 = ax^3 + b$ is y = 4x – 5. **Reason (R):** :Value of $a = 2$ *and* $b = -7$ **Ans : Option(a)**

22.	**Assertion (A):** If $x = at^2$ and $y = 2at$ where 't' is the parameter and 'a' is a constant, then $\frac{d^2y}{dx^2} = \frac{-1}{2at^3}$. **Reason(R):** $\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} \div \frac{d^2x}{dt^2}$
23.	Find the rate of change of volume of a sphere with respect to its surface area when the radius is 4 cm
24.	Find two number whose sum is 24 and whose product is as large as possible.
25.	The demand function of a toy is, $x = 75 - 3p$, where p is selling price of one unit and its total cost function is $TC = 100 + 3x$. For what value of x the profit is maximum.
26.	Find the slopes of the tangent and the normal to the following curves at the indicated points. $x^2 + 3y + y^2 = 5$ at (1,1).
27.	If $x^m . y^n = (x + y)^{(m+n)}$, then show that $\frac{dy}{dx} = \frac{x}{y}$
28.	Find the equation of tangents to the curve $3x^2 - y^2 = 8$ which passes through the point $\left(\frac{4}{3}, 0)\right)$
29.	Find the absolute maximum and absolute minimum values of a function f given by $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval [1,5]
30.	The cost function for x units of a commodity is given by $C(x) = \frac{x^3}{3} + x^2 - 15x + 3$. Find marginal cost function.
31.	A stone is dropped into a quiet lake and waves move in the form of circles at a speed of 4 cm/sec. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?
32.	If price 'p' per unit of an article is $p = 75x - 2x^2$ and the cost function is $C(x) = 350 + 12x + \frac{x^2}{4}$ Find the number of units and the price at which the total profit is maximum. What is the maximum profit.

33.	**4) An architect designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi-circular opening . The window is having a perimeter of 10 meter as shown in the figure.**  **Based on the above information answer the following** **(i) Find the relation between the variables, if $2x$ and $2y$ represents the length and breadth of the rectangular portion of the window.** **(ii) Find the combined area(A) of the rectangular region and semi-circular region of the window expressed as a function of x.** **(iii) Find the length of the rectangular portion of the window should be, if the owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible.** **OR** **Find the maximum area of the whole window.**
34.	**A firm has the cost function $C = \frac{x^3}{3} - 7x^2 + 111x + 50$ and demand function $x = 100 - p$** **(i) Write the total revenue function in terms of x** **(ii) Formulate the total profit function P in terms of x** **(iii) Find the profit maximizing level of output x.** **What is the maximum profit ?**

35.	**Find the equation of the normal to the curve $x^2 + 2y^2 - 4x - 6y + 8 = 0$ at the point whoseabscissa is 2.**